1891MNRAS..52..106J

some faculæ preceding the birth of a spot. And more of the same class of evidence may be found even in the years of greater spot frequency, when the records of their spots and faculæ have been more fully examined and the history of each group is more accurately written.

Stonyhurst Observatory,
Lancashire.

On the Determination of Azimuth by Elongations of Polaris.
By Harold Jacoby, B.A.

When Polaris has been observed within half an hour of elongation for the determination of azimuth, the "reduction to elongation" is frequently computed by the formulæ:

$$m = \frac{2 \sin^2 \frac{1}{2} (t_e - t)}{\sin 1''}$$

$$a_e - a = a_e \,.\, m \,.\, \sin 1''$$

where

t_e, a_e, are the hour angle and azimuth of *Polaris* at elongation,
t, a, " " " " the observation,

a_e and a being reckoned in seconds of arc, from the north point.

But we can avoid the use of the somewhat extended table for m by tabulating $a_e - a$ itself. For if we put

$$M = 6000 \sin 1'' \cdot m$$

$$f = \frac{a_e - 6000}{6000}$$

we shall have

$$a_e - a = M + fM$$

Table I. gives M, with the argument $t_e - t$; and Table II. gives f with the argument a_e. It will be seen that M is really an approximate value of $a_e - a$, and f a correction factor for M. This factor can be multiplied by M with Crelle's Tables, so that logarithms will not be required.

Table III. gives the values of a_e and t_e for all latitudes between 35° and 55°. It is intended to take the place of the logarithmic calculation of a_e and t_e, or at least to serve as a check. The columns headed A and T are computed with the declination 88° 45′ 0″, and furnish approximate values of a_e and t_e. The columns headed x and y give the variations, per second of δ, of the azimuth and hour angle. We shall have, therefore,

$$a_e = A + x(\delta - 88^\circ\, 45')$$

$$t_e = T + y(\delta - 88^\circ\, 45')$$

1891MNRAS..52..106J

where $\delta - 88^\circ\ 45'$ is to be expressed in seconds of arc. These formulæ will give a_e and t_e with all necessary accuracy for many years.

The following observations, made by one of the students of the Summer School in Geodesy of Columbia College, will serve to illustrate the use of the tables. The data employed are—

Station: New London, New Hampshire, No. 3.
Date: 1891 July 13.
Longitude: 71° 59′ West.
Latitude: 43° 24′·0.
Polaris $\alpha = 1^h\ 18^m\ 49^s{\cdot}6$. $\delta = 88^\circ\ 43'\ 23''{\cdot}1$.

We have then

$$\delta - 88^\circ\ 45' = -96''{\cdot}9,$$

and, from Table III.,

$$\text{for } \phi = 43^\circ\ 24'{\cdot}0$$

$$A = \overset{\circ}{1}\ \overset{\prime}{43}\ \overset{\prime\prime}{13}{\cdot}89, \quad x = -1''{\cdot}3766, \quad T = \overset{h}{5}\ \overset{m}{55}\ \overset{s}{16}{\cdot}3, \quad y = +\overset{s}{\cdot}0631$$

$$x(\delta - 88^\circ\ 45') = +2\ 13{\cdot}39, \quad y(\delta - 88^\circ\ 45') = -\ 6{\cdot}1$$

$$a_e = 1\ 45\ 27{\cdot}28, \quad t_e = 5\ 55\ 10{\cdot}2$$

$$\alpha = 1\ 18\ 49{\cdot}6$$

$$\text{Sidereal Time of Elongation} \ \ldots \ \ldots \ \ldots = 19\ 23\ 39{\cdot}4$$

The values here obtained for a_e and t_e are the same as would be given by a logarithmic calculation. The observations of Polaris were (eastern elongation):

No.	Pos.	Sidereal Time.			Corrected Readings.			$t_e - t$		M Table I.	fM Table II.	$a_e - a$	Reduced Readings.		
		h	m	s	°	′	″	m	s	′	″	″	°	′	″
1	D	18	56	1·3	39	37	22·1	27	38·1	43·57	2·40	45·97	39	38	8·1
2	...	18	59	18·8			27·5	24	20·6	33·83	1·86	35·69		38	3·2
3	...	19	9	11·4			41·0	14	28·0	11·95	0·66	12·61		37	53·6
4	...	19	11	20·8			56·0	12	18·6	8·67	0·48	9·15		38	5·2
5	R	19	20	26·3	219	37	43·9	3	13·1	0·59	0·03	0·62	219	37	44·5
6	...	19	23	11·7			38·5	0	27·7	0·01	0·00	0·01			38·5
7	...	19	27	24·4			32·1	3	45·0	0·81	0·04	0·85			33·0
8	...	19	29	17·7			29·2	5	38·3	1·81	0·10	1·91			31·1
9	...	19	33	49·5			33·7	10	10·1	5·90	0·32	6·22			39·9
10	...	19	37	4·0			18·6	13	24·6	10·28	0·57	10·85			29·4
11	...	19	41	0·6			16·8	17	21·2	17·19	0·95	18·14			34·9
12	...	19	45	0·3			7·3	21	20·9	26·02	1·43	27·45			34·8

The above observations are discordant, but they show how the tables are used.

1891MNRAS..52..106J

TABLE I.

M.

m	0^s	10^s	20^s	30^s	40^s	50^s	Difference for 10^s
	″	″	″	″	″	″	″
0	0·00	0·00	0·01	0·01	0·03	0·04	0·01
1	0·06	0·08	0·10	0·13	0·16	0·19	0·03
2	0·23	0·27	0·31	0·36	0·41	0·46	0·05
3	0·51	0·57	0·64	0·70	0·77	0·84	0·07
4	0·91	0·99	1·07	1·16	1·24	1·34	0·09
5	1·43	1·52	1·62	1·73	1·83	1·94	0·10
6	2·06	2·17	2·29	2·41	2·54	2·67	0·12
7	2·80	2·93	3·07	3·21	3·36	3·50	0·14
8	3·66	3·81	3·97	4·13	4·29	4·46	0·16
9	4·63	4·80	4·98	5·15	5·34	5·52	0·18
10	5·71	5·90	6·10	6·30	6·50	6·70	0·20
11	6·91	7·12	7·34	7·55	7·77	8·00	0·22
12	8·22	8·46	8·69	8·92	9·16	9·41	0·24
13	9·65	9·90	10·15	10·41	10·67	10·92	0·26
14	11·19	11·47	11·73	12·00	12·28	12·57	0·28
15	12·85	13·14	13·43	13·72	14·02	14·32	0·29
16	14·62	14·92	15·23	15·54	15·86	16·18	0·31
17	16·50	16·83	17·16	17·49	17·82	18·16	0·33
18	18·50	18·85	19·19	19·54	19·89	20·25	0·35
19	20·61	20·97	21·34	21·71	22·08	22·46	0·37
20	22·83	23·21	23·60	23·99	24·39	24·78	0·39
21	25·18	25·58	25·98	26·39	26·80	27·21	0·41
22	27·64	28·05	28·47	28·90	29·33	29·76	0·43
23	30·19	30·63	31·08	31·52	31·97	32·42	0·45
24	32·88	33·34	33·80	34·26	34·73	35·20	0·46
25	35·67	36·15	36·63	37·11	37·59	38·08	0·48
26	38·58	39·08	39·58	40·08	40·58	41·09	0·50
27	41·60	42·11	42·63	43·15	43·67	41·20	0·52
28	44·73	45·27	45·81	46·34	46·88	47·42	0·54
29	47·97	48·53	49·09	49·65	50·21	50·77	0·56

TABLE II.

f.

a_e Az. at Elong.		$f.$ Corr. Factor.	a_e Az. at Elong.		f Corr. Factor.
°	′		°	′	
1	20	−0·200	2	0	+0·200
1	30	−0·100	2	10	+0·300
1	40	0·000	2	20	+0·400
1	50	+0·100			

1891MNRAS..52..106J

TABLE III.

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T. Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ″	° ′ ″	″	h m s	s
35 0	1 31 33·70	−1·2210	5 56 29·9	+·0467
	11·24	24	1·3	3
10	1 31 44·94	−1·2234	5 56 28·6	+·0470
	11·33	26	1·3	3
20	1 31 56·27	−1·2260	5 56 27·3	+·0473
	11·43	25	1·3	3
30	1 32 7·70	−1·2285	5 56 26·0	+·0476
	11·52	26	1·3	3
40	1 32 19·22	−1·2311	5 56 24·7	+·0479
	11·61	25	1·4	3
50	1 32 30·83	−1·2336	5 56 23·3	+·0482
	11·71	26	1·3	3
36 0	1 32 42·54	−1·2362	5 56 22·0	+·0485
	11·81	27	1·3	3
10	1 32 54·35	−1·2389	5 56 20·7	+·0488
	11·90	26	1·4	3
20	1 33 6·25	−1·2415	5 56 19·3	+·0491
	12·01	27	1·4	3
30	1 33 18·26	−1·2442	5 56 17·9	+·0494
	12·10	27	1·3	3
40	1 33 30·36	−1·2469	5 56 16·6	+·0497
	12·20	27	1·3	3
50	1 33 42·56	−1·2496	5 56 15·3	+·0500
	12·31	27	1·4	3
37 0	1 33 54·87	−1·2523	5 56 13·9	+·0503
	12·40	28	1·4	3
10	1 34 7·27	−1·2551	5 56 12·5	+·0506
	12·51	28	1·4	3
20	1 34 19·78	−1·2579	5 56 11·1	+·0509
	12·61	28	1·4	3
30	1 34 32·39	−1·2607	5 56 9·7	+·0512
	12·73	28	1·4	3
40	1 34 45·12	−1·2635	5 56 8·3	+·0515
	12·81	28	1·3	3
50	1 35 57·93	−1·2663	5 56 7·0	+·0518
	12·93	29	1·4	3
38 0	1 35 10·86	−1·2692	5 56 5·6	+·0521
	13·04	29	1·5	3
10	1 35 23·90	−1·2721	5 56 4·1	+·0524
	13·14	29	1·4	3
20	1 35 37·04	−1·2750	5 56 2·7	+·0527
	13·25	30	1·4	4
30	1 35 50·29	−1·2780	5 56 1·3	+·0531
	13·37	29	1·4	3
40	1 36 3·66	−1·2809	5 55 59·9	+·0534
	13·47	31	1·4	3
50	1 36 17·13	−1·2840	5 55 58·5	+·0537
	13·59	30	1·5	3

Azimuth at Elongation $= \mathrm{A} + x(\delta - 88^\circ\ 45')$. Hour Angle at Elongation $= \mathrm{T} + y(\delta - 88^\circ\ 45')$. $(\delta - 88^\circ\ 45')$ to be expressed in seconds of arc.

1891MNRAS..52..106J

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
39 0	1 36 30·72	−1·2870	5 55 57·0	+·0540
	13·70	30	1·5	4
10	1 36 44·42	−1·2900	5 55 55·5	+·0544
	13·82	31	1·4	3
20	1 36 58·24	−1·2931	5 55 54·1	+·0547
	13·92	31	1·4	3
30	1 37 12·16	−1·2962	5 55 52·7	+·0550
	14·05	31	1·5	3
40	1 37 26·21	−1·2993	5 55 51·2	+·0553
	14·17	32	1·5	3
50	1 37 40·38	−1·3025	5 55 49·7	+·0556
	14·28	32	1·5	4
40 0	1 37 54·66	−1·3057	5 55 48·2	+·0560
	14·40	32	1·5	3
10	1 38 9·06	−1·3089	5 55 46·7	+·0663
	14·53	32	1·5	3
20	1 38 23·59	−1·3121	5 55 45·2	+·0566
	14·65	32	1·5	4
30	1 38 38·24	−1·3153	5 55 43·7	+·0570
	14·76	33	1·5	3
40	1 38 53·00	−1·3186	5 55 42·2	+·0573
	14·90	33	1·5	3
50	1 39 7·90	−1·3219	5 55 40·7	+·0576
	15·02	34	1·6	4
41 0	1 39 22·92	−1·3253	5 55 39·1	+·0580
	15·14	33	1·5	3
10	1 39 38·06	−1·3286	5 55 37·6	+·0583
	15·28	34	1·5	4
20	1 39 53·34	−1·3320	5 55 36·1	+·0587
	15·40	34	1·6	3
30	1 40 8·74	−1·3354	5 55 34·5	+·0590
	15·53	35	1·6	4
40	1 40 24·27	−1·3389	5 55 32·9	+·0594
	15·67	35	1·5	3
50	1 40 39·94	−1·3424	5 55 31·4	+·0597
	15·80	35	1·6	4
42 0	1 40 55·74	−1·3459	5 55 29·8	+·0601
	15·93	36	1·6	3
10	1 41 11·67	−1·3495	5 55 28·2	+·0604
	16·07	35	1·5	4
20	1 41 27·74	−1·3530	5 55 26·7	+·0608
	16·20	36	1·6	3
30	1 41 43·94	−1·3566	5 55 25·1	+·0611
	16·35	37	1·7	4
40	1 42 0·29	−1·3603	5 55 23·4	+·0615
	16·48	36	1·6	3
50	1 42 16·77	−1·3639	5 55 21·8	+·0618
	16·63	37	1·6	4

Azimuth at Elongation $= \mathrm{A} + x(\delta - 88^\circ\ 45')$. Hour Angle at Elongation $= \mathrm{T} + y(\delta - 88^\circ\ 45')$. $(\delta - 88^\circ\ 45')$ to be expressed in seconds of arc.

1891MNRAS..52..106J

φ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
43 0	1 42 33·40	−1·3676	5 55 20·2	+·0622
	16·77	37	1·7	4
10	1 42 50·17	−1·3713	5 55 18·5	+·0626
	16·91	38	1·6	3
20	1 43 7·08	−1·3751	5 55 16·9	+·0629
	17·06	38	1·6	4
30	1 43 24·14	−1·3789	5 55 15·3	+·0633
	17·20	38	1·7	4
40	1 43 41·34	−1·3827	5 55 13·6	+·0637
	17·36	39	1·7	3
50	1 43 58·70	−1·3866	5 55 11·9	+·0640
	17·50	39	1·7	4
44 0	1 44 16·20	−1·3905	5 55 10·2	+·0644
	17·65	39	1·7	4
10	1 44 33·85	−1·3944	5 55 8·5	+·0648
	17·81	40	1·7	4
20	1 44 51·66	−1·3984	5 55 6·8	+·0652
	17·97	40	1·7	4
30	1 45 9·63	−1·4024	5 55 5·1	+·0656
	18·12	40	1·7	4
40	1 45 27·75	−1·4064	5 55 3·4	+·0660
	18·28	41	1·7	3
50	1 45 46·03	−1·4105	5 55 1·7	+·0663
	18·44	41	1·8	4
45 0	1 46 4·47	−1·4146	5 54 59·9	+·0667
	18·60	41	1·7	4
10	1 46 23·07	−1·4187	5 54 58·2	+·0671
	18·76	42	1·8	4
20	1 46 41·83	−1·4229	5 54 56·4	+·0675
	18·93	42	1·7	4
30	1 47 0·76	−1·4271	5 54 54·7	+·0679
	19·10	42	1·8	4
40	1 47 19·86	−1·4313	5 54 52·9	+·0683
	19·26	43	1·8	4
50	1 47 39·12	−1·4356	5 54 51·1	+·0687
	19·44	43	1·8	4
46 0	1 47 58·56	−1·4399	5 54 49·3	+·0691
	19·60	44	1·8	4
10	1 48 18·16	−1·4443	5 54 47·5	+·0695
	19·79	44	1·9	4
20	1 48 37·95	−1·4487	5 54 45·6	+·0699
	19·95	44	1·8	4
30	1 48 57·90	−1·4531	5 54 43·8	+·0703
	20·14	45	1·9	4
40	1 49 18·04	−1·4576	5 54 41·9	+·0707
	20·32	45	1·8	4
50	1 49 38·36	−1·4621	5 54 40·1	+·0711
	20·50	46	1·9	4

Azimuth at Elongation $= \text{A} + x(\delta - 88^\circ\ 45')$. Hour Angle at Elongation $= \text{T} + y(\delta - 88^\circ\ 45')$. $(\delta - 88^\circ\ 45')$ to be expressed in seconds of arc.

1891MNRAS..52..106J

φ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
47 0	1 49 58·86	−1·4667	5 54 38·2	+·0715
	20·68	46	1·9	5
10	1 50 19·54	−1·4713	5 54 36·3	+·0720
	20·87	46	1·9	4
20	1 50 40·41	−1·4759	5 54 34·4	+·0724
	21·06	47	1·9	4
30	1 51 1·47	−1·4806	5 54 32·5	+·0728
	21·25	48	1·9	4
40	1 51 22·72	−1·4854	5 54 30·6	+·0732
	21·45	47	1·9	4
50	1 51 44·17	−1·4901	5 54 28·7	+·0736
	21·63	48	2·0	5
48 0	1 52 5·80	−1·4949	5 54 26·7	+·0741
	21·84	49	1·9	4
10	1 52 27·64	−1·4998	5 54 24·8	+·0745
	22·03	49	2·0	5
20	1 52 49·67	−1·5047	5 54 22·8	+·0750
	22·24	49	2·0	4
30	1 53 11·91	−1·5096	5 54 20·8	+·0754
	22·44	50	2·0	4
40	1 53 34·35	−1·5146	5 54 18·8	+·0758
	22·65	51	2·0	5
50	1 53 57·00	−1·5197	5 54 16·8	+·0763
	22·86	51	2·0	4
49 0	1 54 19·86	−1·5248	5 54 14·8	+·0767
	23·07	51	2·1	5
10	1 54 42·93	−1·5299	5 54 12·7	+·0772
	23·28	52	2·0	4
20	1 55 6·21	−1·5351	5 54 10·7	+·0776
	23·51	52	2·1	5
30	1 55 29·72	−1·5403	5 54 8·6	+·0781
	23·71	53	2·1	5
40	1 55 53·43	−1·5456	5 54 6·5	+·0786
	23·95	53	2·0	4
50	1 56 17·38	−1·5509	5 54 4·5	+·0790
	24·17	54	2·1	5
50 0	1 56 41·55	−1·5563	5 54 2·4	+·0795
	24·39	54	2·1	5
10	1 57 5·94	−1·5617	5 54 0·3	+·0800
	24·63	55	2·2	4
20	1 57 30·57	−1·5672	5 53 58·1	+·0804
	24·86	55	2·2	5
30	1 57 55·43	−1·5727	5 53 55·9	+·0809
	25·09	56	2·1	5
40	1 58 20·52	−1·5783	5 53 53·8	+·0814
	25·34	56	2·2	5
50	1 58 45·86	−1·5839	5 53 51·6	+·0819
	25·58	57	2·2	5

Azimuth at Elongation $= A + x(\delta - 88° 45')$. Hour Angle at Elongation $= T + y(\delta - 88° 45')$. $(\delta - 88° 45')$ to be expressed in seconds of arc.

1891MNRAS..52..106J

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
51 0	1 59 11·44	−1·5896	5 53 49·4	+·0824
	25·83	57	2·2	5
10	1 59 37·27	−1·5953	5 53 47·2	+·0829
	26·06	58	2·2	5
20	2 0 3·33	−1·6011	5 53 45·0	+·0834
	26·32	59	2·3	5
30	2 0 29·65	−1·6070	5 53 42·7	+·0839
	26·58	59	2·2	5
40	2 0 56·23	−1·6129	5 53 40·5	+·0844
	26·83	60	2·3	5
50	2 1 23·06	−1·6189	5 53 38·2	+·0849
	27·10	60	2·3	5
52 0	2 1 50·16	−1·6249	5 53 35·9	+·0854
	27·36	61	2·3	5
10	2 2 17·52	−1·6310	5 53 33·6	+·0859
	27·63	62	2·3	5
20	2 2 45·15	−1·6372	5 53 31·3	+·0864
	27·90	62	2·4	6
30	2 3 13·05	−1·6434	5 53 28·9	+·0870
	28·18	63	2·4	5
40	2 3 41·23	−1·6497	5 53 26·5	+·0875
	28·46	63	2·4	5
50	2 4 9·69	−1·6560	5 53 24·1	+·0880
	28·74	64	2·4	6
53 0	2 4 38·43	−1·6624	5 53 21·7	+·0886
	29·02	64	2·4	5
10	2 5 7·45	−1·6688	5 53 19·3	+·0891
	29·32	65	2·4	5
20	2 5 36·77	−1·6753	5 53 16·9	+·0896
	29·60	66	2·4	6
30	2 6 6·37	−1·6819	5 53 14·5	+·0902
	29·91	67	2·5	5
40	2 6 36·28	−1·6886	5 53 12·0	+·0907
	30·21	67	2·5	6
50	2 7 6·49	−1·6953	5 53 9·5	+·0913
	30·52	68	2·6	5
54 0	2 7 37·01	−1·7021	5 53 6·9	+·0918
	30·82	69	2·5	6
10	2 8 7·83	−1·7090	5 53 4·4	+·0924
	31·15	69	2·5	
20	2 8 38·98	−1·7159	5 53 1·9	+·0930
	31·46	70	2·6	5
30	2 9 10·44	−1·7229	5 52 59·3	+·0935
	31·78	70	2·6	6
40	2 9 42·22	−1·7299	5 52 56·7	+·0941
	32·11	72	2·6	6
50	2 10 14·33	−1·7371	5 52 54·1	+·0947
	32·45	74	2·7	6
55 0	2 10 46·78	−1·7443	5 52 51·4	+·0953

Azimuth at Elongation = A + $x(\delta - 88° 45')$. Hour Angle at Elongation = T + $y(\delta - 88° 45')$. $(\delta - 88° 45')$ to be expressed in seconds of arc.

Columbia College, New York,
1891 *September* 30.

1891MNRAS..52..106J

On the Reduction of Transit Observations by the Method of Least Squares. By Harold Jacoby, B.A.

When it is desired to adjust a series of observations by the method of least squares, the process usually employed is not always the most advantageous one available—in fact, the usual method is really an ingenious way of finding the *adjusted* values of the unknowns from the coefficients and absolute terms of the original observation equations. Occasionally, however, these coefficients are themselves all functions of some single quantity. In such a case there must exist a functional relation between the adjusted values of the unknowns and the single quantity in question; and this relation may possibly be evaluated *without computing the actual coefficients of the observation equations.* The above applies to the transit instrument in the meridian, for the coefficients of the observation equations, as well as the weight-factor, are then all functions of the declination. The *latitude* affects them as a constant only.

Before going into the details of the application of the above remarks to actual work, it is desirable to call attention to a theorem in the method of least squares which is very often of considerable use. Let there be given two groups of linear observation equations of the form:

$$\begin{array}{ll} a_1w+b_1y+c_1z+d_1=0, & a_1'x+b_1'y+c_1'z+d_1'=0, \\ a_2w+b_2y+c_2z+d_2=0, & a'_2x+b_2'y+c_2'z+d_2'=0, \\ \cdots\cdots & \cdots\cdots \end{array}$$

in which it will be seen that the equations of the first group lack the term in x, and those of the second group lack the term in w. Now let normal equations be formed from each group as follows:

$$\begin{array}{rr} [aa]w+[ab]y+[ac]z+[ad]=0, & [a'a']x+[a'b']y+[a'c']z+[a'd']=0. \\ [bb]y+[bc]z+[bd]=0, & [b'b']y+[b'c']z+[b'd']=0. \\ [cc]z+[cd]=0, & [c'c']z+[c'd']=0, \\ [dd] & [d'd']. \end{array}$$

Let w and x be eliminated in the usual way, giving:

$$\begin{array}{rr} [bb\,.\,1]y+[bc\,.\,1]z+[bd\,.\,1]=0, & [b'b'\,.\,1]y+[b'c'\,.\,1]z+[b'd'\,.\,1]=0, \\ [cc\,.\,1]z+[cd\,.\,1]=0, & [c'c'\,.\,1]z+[c'd'\,.\,1]=0, \\ [dd\,.\,1] & [d'd'\,.\,1]. \end{array}$$

Now let these equations be added, term by term, giving:

$$\begin{array}{r} ([bb\,.\,1]+[b'b'\,.\,1])y+([bc\,.\,1]+[b'c'\,.\,1])z+([bd\,.\,1]+[b'd'\,.\,1])=0, \\ ([cc\,.\,1]+[c'c'\,.\,1])z+([cd\,.\,1]+[c'd'\,.\,1])=0, \\ ([dd\,.\,1]+[d'd'\,.\,1]); \end{array}$$